CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions - Exercise 1.1

1. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set A = {1, 2, 3, ... 13, 14} defined as R = $\{(x, y): 3x - y = 0\}$ **.**

(ii) Relation R in the set N of natural numbers defined as R = {(x, y) : y = x + 5 and x < 4}

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as R = $\{(x, y): y \text{ is divisible by } x\}$.

(iv) Relation R in the set Z of all integers defined as R = $\{(x, y): x - y \text{ is an integer}\}$.

(v) Relation R in the set A of human beings in a town at a particular time given by:

(a) R = $\{(x, y): x \text{ and } y \text{ work at the same place}\}$.

(b) R = $\{(x, y): x \text{ and } y \text{ live in the same locality}\}$.

(c) R = $\{(x, y): x \text{ is exactly 7 cm taller than } y\}$.

(d) R = $\{(x, y): x \text{ is wife of } y\}$.

(e) R = $\{(x, y): x \text{ is father of } y\}$.

Ans. (i) R = $\{(x, y): 3x - y = 0\}$, in A = {1, 2, 3, 4, 5, 6, ...13, 14}

Clearly R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Since, $(x, x) \notin$ R, $\therefore$ R is not reflexive.

Again $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also (1, 3) $\in$ R and (3, 9) $\in$ R but (1, 9) $\notin$ R, $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(ii) R = $\{(x, y) : y = x + 5 \text{ and } x < 4\}$ in set N of natural numbers.

Clearly R = {(1, 6), (2, 7), (3, 8)}

Now $(x, x) \notin$ R, $\therefore$ R is not reflexive.

Again $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also (1, 6) $\in$ R and (2, 7) $\in$ R but (1, 7) $\notin$ R, $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(iii) R = $\{(x, y) : y \text{ is divisible by } x\}$ in A = {1, 2, 3, 4, 5, 6}

Clearly R = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (5, 5), (6, 6)

Now (x, x) i.e., (1, 1), (2, 2) and (3, 3) $\in$ R $\therefore$ R is reflexive.

Again (x, y) i.e., (1, 2) $\in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also (1, 4) $\in$ R and (4, 4) $\in$ R and (1, 4) $\in$ R, $\therefore$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

(iv) R = $\{(x, y) : x - y \text{ is an integer}\}$ in set Z of all integers.

Now (x, x) i.e., (1, 1) = 1 – 1 = 0 $\in$ Z $\therefore$ R is reflexive.

Again (x, y) $\in$ R and $(y, x) \in$ R, i.e., $x - y$ and $y - x$ are an integer $\therefore$ R is symmetric.

Also $(x_1, y_1) = x_1 - y_1$ $\in$ Z and $(y_1, z_1) = y_1 - z_1$ $\in$ Z and

(x_1, z_1) $\in$ R, $\therefore$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(v) Relation R in the set A of human being in a town at a particular time.

(a) R = $\{(x, y): x \text{ and } y \text{ work at the same place}\}$

Since $(x, x) \in$ R, because x and x work at the same place. $\therefore$ R is reflexive.

Now, if $(x, y) \in$ R and $(x, y) \in$ R, since x and y work at the same place and y and x work at the same place. $\therefore$ R is symmetric.

Now, if $(x, y) \in$ R and $(y, z) \in$ R $\Rightarrow$ $(x, z) \in$ R. $\therefore$ R is transitive

Therefore, R is reflexive, symmetric and transitive.

(b) R = $\{(x, y): x \text{ and } y \text{ live in the same locality}\}$

Since $(x, x) \in$ R, because x and x live in the same locality. $\therefore$ R is reflexive.

Also $(x, y) \in$ R $\Rightarrow (y, x) \in$ R because x and y live in same locality and y and x also live in same locality. $\therefore$ R is symmetric.

Again $(x, y) \in$ R and $(y, z) \in$ R $\Rightarrow (x, z) \in$ R $\therefore$ R is transitive.

Therefore, R is reflexive, symmetric and transitive.

(c) R = $\{(x, y): x \text{ is exactly 7 cm taller than } y\}$

x is not exactly 7 cm taller than x, so $(x, x) \notin$ R $\therefore$ R is not reflexive.

Also x is exactly 7 cm taller than y but y is not 7 cm taller than x, so $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Now x is exactly 7 cm taller than y and y is exactly 7 cm taller than z then it does not imply that x is exactly 7 cm taller than z. $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(d) R = $\{(x, y) : x \text{ is wife of } y\}$

x is not wife of x, so $(x, x) \notin$ R $\therefore$ R is not reflexive.

Also x is wife of y but y is not wife of x, so $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also $(x, y) \in$ R and $(y, z) \in$ R then $(x, z) \notin$ R $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

(e) R = $\{(x, y) : x \text{ is father of } y\}$

x is not father of x, so $(x, x) \notin$ R $\therefore$ R is not reflexive.

Also x is father of y but y is not father of x,

so $(x, y) \in$ R but $(y, x) \notin$ R $\therefore$ R is not symmetric.

Also $(x, y) \in$ R and $(y, z) \in$ R then $(x, z) \notin$ R $\therefore$ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

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2. Show that the relation R in the set R of real numbers defined as R = $\{(a, b) : a \le b^2\}$ **is neither reflexive nor symmetric nor transitive.**

Ans. R = $\{(a, b) : a \le b^2\}$, Relation R is defined as the set of real numbers.

(i) Whether $(a, a) \in$ R, then $a \le a^2$ which is false. $\therefore$ R is not reflexive.

(ii) Whether $(a, b) = (b, a)$, then $a \le b^2$ and $b \le a^2$, it is false. $\therefore$ R is not symmetric.

(iii) Now $a \le b^2$, $b \le c^2 \Rightarrow a \le c^4$, which is false. $\therefore$ R is not transitive

Therefore, R is neither reflexive, nor symmetric and nor transitive.

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3. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = $\{(a,b): b=a+1\}$ is reflexive, symmetric or transitive.

Ans. R = $\{(a,b): b=a+1\}, a, b \in$ R

Now R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)} and $b=a+1$

(i) When $b=a \Rightarrow a=a+1$, which is false, so $(a,a) \notin$ R, ∴ R is not reflexive.

(ii) Whether $(a,b)=(b,a)$, then $b=a+1$ and $a=b+1$, false ∴ R is not symmetric.

(iii) Now if $(a,b) \in$ R, $(b,c) \in$ R $\Rightarrow (a,c) \in$ R

(iv) Then $b=a+1$ and $c=b+1 \Rightarrow c=a+2$ which is false. ∴ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

4. Show that the relation R in R defined as R = $\{(a,b): a \le b\}$, is reflexive and transitive but not symmetric.

Ans. (i) $a \le a$ which is true, so $(a,a) \in$ R, ∴ R is reflexive.

(ii) $a \le b$ but $b \le a$ which is false. ∴ R is not symmetric.

(iii) $a \le b$ and $b \le c \Rightarrow a \le c$ which is true. ∴ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

5. Check whether the relation R in R defined by R = $\{(a,b): a \le b^3\}$ is reflexive, symmetric or transitive.

Ans. (i) For (a,a), $a < a^3$ which is false. ∴ R is not reflexive.

(ii) For $(a,b), a < b^3$ and $(b,a), b < a^3$ which is false. ∴ R is not symmetric.

(iii) For (a,b), (b,c) and (a,c), $a < a^9$ which is false. ∴ R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

6. Show that the relation in the set A = {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Ans. R = {(1, 2), (2, 1)}, so for (a, a), (1, 1) $\notin$ R. $\therefore$ R is not reflexive.

Also if $(a, b) \in$ then $(b, a) \in$ R $\therefore$ R is symmetric.

Now $(a, b) \in$ R and $(b, c) \in$ then does not imply $(a, c) \notin$ R $\therefore$ R is not transitive..

Therefore, R is symmetric but neither reflexive nor transitive.

7. Show that the relation R in the set A of all the books in a library of a college, given by R = $\{(x, y) : x \text{ and } y \text{ have same number of pages}\}$ is an equivalence relation.

Ans. Books x and x have same number of pages $\Rightarrow$ $(x, x) \in$ R $\therefore$ R is reflexive.

If $(x, y) \in$ R $\Rightarrow$ $(y, x) \in$ R, so $(x, y) = (y, x)$ $\therefore$ R is symmetric.

Now if $(x, y) \in$ R, $(y, z) \in$ R $\Rightarrow$ $(x, z) \in$ R $\therefore$ R is transitive.

Since R is reflexive, symmetric and transistive, therefore, R is an equivalence relation.

8. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = $\{(a, b) : |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

Ans. A = {1, 2, 3, 4, 5} and R = $\{(a, b) : |a-b| \text{ is even}\}$, then R = {(1, 3), (1, 5), (3, 5), (2, 4)}

(a) For (a, a), $|a-a| = 0$ which is even. $\therefore$ R is reflexive.

If $|a-b|$ is even, then $|b-a|$ is also even. $\therefore$ R is symmetric.

Now, if $|a-b|$ and $|b-c|$ is even then $|a-b+b-c|$ $\Rightarrow$ $|a-c|$ is also even. $\therefore$ R is transitive.

Therefore, R is an equivalence relation.

(b) Elements of {1, 3, 5} are related to each other.

Since $|1-3|=2$, $|3-5|=2$, $|1-5|=4$, all are even numbers

$\Rightarrow$ Elements of {1, 3, 5} are related to each other.

Similarly elements of (2, 4) are related to each other.

Since $|2-4|=2$ an even number, then no element of the set {1, 3, 5} is related to any element of (2, 4).

Hence no element of {1, 3, 5} is related to any element of {2, 4}.

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9. Show that each of the relation R in the set A = $\{x \in Z : 0 \le x \le 12\}$, **given by:**

(i) R = $\{(a, b) : |a-b| \text{ is a multiple of } 4\}$

(ii) R = $\{(a, b) : a = b\}$ is an equivalence relation. Find the set of all elements related to 1 in each case.

Ans. A = { x ∈ Z : 0 ≤ x ≤ 12} = {0,1,2,3,4,5,6,7,8,9,10,11,12}

(i) R = { (a,b) : |a - b| is a multiple of 4}
For any element a ∈ A, we have (a, a) ∈ R as |a - a = 0| is a multiple of 4.
∴ R is reflexive.
Now, let (a, b) ∈ R ⇒ |a - b| is a multiple of 4.
⇒ |-(a - b)| ⇒ |b - a| is a multiple of 4.
⇒ (b, a) ∈ R
∴ R is symmetric.
Now, let (a, b), (b, c) ∈ R.
⇒ |(a - b)| is a multiple of 4 and |(b - c)| is a multiple of 4.

$\Rightarrow$ (a - b) is a multiple of 4 and (b - c) is a multiple of 4.

$\Rightarrow$ (a - c) = (a – b) + (b – c) is a multiple of 4.

$\Rightarrow$ $|a - c|$ is a multiple of 4.

$\Rightarrow (a, c) \in R$

$\therefore$ R is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is {1, 5, 9} since

$|1 - 1| = 0$ is a multiple of 4,

$|5 - 1| = 4$ is a multiple of 4, and

$|9 - 1| = 8$ is a multiple of 4.

(ii) $R = \{(a, b): a = b\}$

For any element $a \in A$, we have $(a, a) \in R$, since $a = a$.

$\therefore$ R is reflexive.

Now, let $(a, b) \in R$.

$\Rightarrow a = b$

$\Rightarrow b = a$

$\Rightarrow (b, a) \in R$

$\therefore$ R is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow a = b$ and $b = c$

$\Rightarrow a = c$

$\Rightarrow (a, c) \in R$

$\therefore$ R is transitive.

Hence, R is an equivalence relation.

The elements in R that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is {1}.

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10. Give an example of a relation, which is:

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

Ans. (i) The relation "is perpendicular to" l_1 is not perpendicular to l_2.

If $l_1 \perp l_2$ then $l_2 \perp l_1$, however if $l_1 \perp l_2$ and $l_2 \perp l_3$ then l_1 is not perpendicular to l_3.

So it is clear that R "is perpendicular to" is a symmetric but neither reflexive nor transitive.

(ii) Relation R = $\{(x, y) : x > y\}$

We know that $x > x$ is false. Also $x > y$ but $y > x$ is false and if $x > y$, $y > z$ this implies $x > z$.

Therefore, R is transitive, but neither reflexive nor symmetric.

(iii) "is friend of" R = $\{(x, y) : x \text{ is a friend of } y\}$

It is clear that x is friend of x. $\therefore$ R is reflexive.

Also x is friend of y and y is friend of x. $\therefore$ R is symmetric.

Also if x is friend of y and y is friend of z then

x cannot be friend of z $\therefore$ R is not transitive.

Therefore, R is reflexive and symmetric but not transitive.

(iv) "is greater or equal to" R = $\{(x, y) : x \geq y\}$

It is clear that $x \geq x$ $\therefore$ R is reflexive.

And $x \geq y$ does not imply $y \geq x$ $\therefore$ R is not symmetric.

But $x \geq y$, $y \geq z \Rightarrow x \geq z$ $\therefore$ R is transitive.

Therefore, R is reflexive and transitive but not symmetric.

(v) “is brother of” R = $\{(x, y) : x \text{ is a brother of } y\}$

It is clear that x **is not the brother of** x. $\therefore$ **R is not reflexive.**

Also x **is brother of** y **and** y **is brother of** x. $\therefore$ **R is symmetric.**

Also if x **is brother of** y **and** y **is brother of** z **then**

x **can be brother of** z $\therefore$ **R is transitive.**

Therefore, R is symmetric and transitive but not reflexive.

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11. Show that the relation R in the set A of points in a plane given by R = {(P. Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P $\neq$ **(0, 0) is the circle passing through P with origin as centre.**

Ans. Part I: R = {(P, Q): distance of the point P from the origin is the same as the distance of the point Q from the origin}

Let P(x_1, y_1) **and Q**(x_2, y_2) **and O (0, 0).**

$\therefore$ **OP = OQ** $\Rightarrow$ $\sqrt{x_1^2 + y_1^2} = \sqrt{x_2^2 + y_2^2} \Rightarrow x_1^2 + y_1^2 = x_2^2 + y_2^2$

Now, For (P, P), OP = OP $\therefore$ **R is reflexive.**

Also OP = OQ and OQ = OP $\Rightarrow$ **(P, Q) = (Q, P)**$\in$ **R** $\therefore$ **R is symmetric.**

Also OP = OQ and OQ = OR $\Rightarrow$ **OP = OQ** $\therefore$ **R is transitive.**

Therefore, R is an equivalent relation.

Part II: As $x_1^2 + y_1^2 = x_2^2 + y_2^2 = r^2$ **(let)** $\Rightarrow$ $x^2 + y^2 = r^2$ **which represents a circle with centre (0, 0) and radius** r.

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12. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is an equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Ans. Part I: R = {(T_1, T_2): T_1 is similar to T_2} and T_1, T_2 are triangle.

We know that each triangle similar to itself and thus $(T_1, T_2) \in R$ $\therefore$ R is reflexive.

Also two triangles are similar, then $T_1 \cong T_2 \Rightarrow T_1 \cong T_2$ $\therefore$ R is symmetric.

Again, if then $T_1 \cong T_2$ and then $T_2 \cong T_3 \Rightarrow$ then $T_1 \cong T_3$ $\therefore$ R is transitive.

Therefore, R is an equivalent relation.

Part II: It is given that T_1, T_2 and T_3 are right angled triangles.

$\Rightarrow T_1$ with sides 3, 4, 5, T_2 with sides 5, 12, 13 and

T_3 with sides 6, 8, 10

Since, two triangles are similar if corresponding sides are proportional.

Therefore, $\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$

Therefore, T_1 and T_3 are related.

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13. Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2) : P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4, and 5?

Ans. Part I: R = {(P_1, P_2): P_1 and P_2 have same number of sides}

(i) Consider the element (P_1, P_2), it shows P_1 and P_2 have same number of sides. Therefore, R is reflexive.

(ii) If $(P_1, P_2) \in R$ then also $(P_2, P_1) \in R$

$\therefore$ $(P_1, P_2) = (P_2, P_1)$ as P_1 and P_2 have same number of sides, therefore, R is symmetric.

(iii) If $(P_1, P_2) \in R$ and $(P_2, P_3) \in R$ then also $(P_1, P_3) \in R$ as P_1, P_2 and P_3 have same number of sides, therefore, R is transitive.

Therefore, R is an equivalent relation.

Part II: we know that if 3, 4, 5 are the sides of a triangle, then the triangle is right angled triangle. Therefore, the set A is the set of right angled triangle.

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14. Let L be the set of all lines in XY plane and R be the relation in L defined as R = $\{(L_1, L_2) : L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Ans. Part I: R = $\{(L_1, L_2): L_1$ is parallel to $L_2\}$

(i) It is clear that $L_1 \parallel L_1$ i.e., $(L_1, L_1) \in R$ $\therefore$ R is reflexive.

(ii) If $L_1 \parallel L_2$ and $L_2 \parallel L_1$ then $(L_1, L_2) \in R$ $\therefore$ R is symmetric.

(iii) If $L_1 \parallel L_2$ and $L_2 \parallel L_3 \Rightarrow L_1 \parallel L_3$ $\therefore$ R is transitive.

Therefore, R is an equivalent relation.

Part II: All the lines related to the line $y = 2x + 4$ and $y = 2x + k$ where k is a real number.

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15. Let R be the relation in the set {1, 2, 3, 4} given by R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer:

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Ans. Let R be the relation in the set {1, 2, 3, 4} is given by

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}

(a) (1, 1), (2, 2), (3, 3), (4, 4) $\in$ R $\therefore$ R is reflexive.

(b) (1, 2) $\in$ R but (2, 1) $\notin$ R $\therefore$ R is not symmetric.

(c) If (1, 3) $\in$ R and (3, 2) $\in$ R then (1, 2) $\in$ R $\therefore$ R is transitive.

Therefore, option (B) is correct.

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16. Let R be the relation in the set N given by R = $\{(a, b): a = b - 2, b > 6\}$. Choose the correct answer:

(A) (2, 4) $\in$ R

(B) (3, 8) $\in$ R

(C) (6, 8) $\in$ R

(D) (8, 7) $\in$ R

Ans. Given: $a = b - 2, b > 6$

(A) $a = 2, b = 4$, Here $b > 6$ is not true, therefore, this option is incorrect.

(B) $a = 3, b = 8$ and $a = b - 2 \Rightarrow 3 = 8 - 2 \Rightarrow$ 3 = 6, which is false.

Therefore, this option is incorrect.

(C) $a = 6, b = 8$ and $a = b - 2 \Rightarrow 6 = 8 - 2 \Rightarrow$ 6 = 6, which is true.

Therefore, option (C) is correct.

(D) $a = 8, b = 7$ and $a = b - 2 \Rightarrow 8 = 7 - 2 \Rightarrow$ 8 = 5, which is false.

CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions - Exercise 1.2

1. Show that the function $f: \mathbb{R}_* \to \mathbb{R}_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where $\mathbb{R}_*$ is the set of all non-zero real numbers. Is the result true, if the domain $\mathbb{R}_*$ is replaced by N with co-domain being same as $\mathbb{R}_*$?

Ans. $f(x) = \frac{1}{x}, f: \mathbb{R}_* \to \mathbb{R}_*$

Part I: $f(x_1) = \frac{1}{x_1}$ and $f(x_2) = \frac{1}{x_2}$

If $f(x_1) = f(x_2)$ then $\frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

$f(x) = \frac{1}{x}$

$\Rightarrow y = \frac{1}{x}$

$\Rightarrow x = \frac{1}{y}$

$\Rightarrow f\left(\frac{1}{y}\right) = y$ $\therefore$ f is onto.

Part II: When domain R is replaced by N, co-domain R remaining the same, then,

$f = \mathrm{N} \to \mathrm{R}$

If $f(x_1) = f(x_2)$

$\Rightarrow \dfrac{1}{n_1} = \dfrac{1}{n_2}$

$\Rightarrow n_1 = n_2$ where $n_1, n_2 \in \mathrm{N}$

$\therefore$ f is one-one.

But every real number belonging to co-domain may not have a pre-image in N.

e.g. $\dfrac{1}{3} = \dfrac{3}{2} \neq \mathrm{N}$ $\therefore$ f is not onto.

2. Check the injectivity and surjectivity of the following functions:

(i) $f: \mathrm{N} \to \mathrm{N}$ **given by** $f(x) = x^2$

(ii) $f: \mathrm{Z} \to \mathrm{Z}$ **given by** $f(x) = x^2$

(iii) $f: \mathrm{R} \to \mathrm{R}$ **given by** $f(x) = x^2$

(iv) $f: \mathrm{N} \to \mathrm{N}$ **given by** $f(x) = x^3$

(v) $f: \mathrm{Z} \to \mathrm{Z}$ **given by** $f(x) = x^3$

Ans. (i) $f: \mathrm{N} \to \mathrm{N}$ given by $f(x) = x^2$

If $f(x_1) = f(x_2)$ then $x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is injective.

There are such numbers of co-domain which have no image in domain N.

e.g. 3 $\in$ co-domain N, but there is no pre-image in domain of f.

therefore f is not onto. $\therefore$ f is not surjective.

(ii) $f: Z \to Z$ given by $f(x) = x^2$

Since, Z = $\{0, \pm1, \pm2, \pm3, \ldots\ldots\}$ therefore, $f(-1) = f(1) = 1$

$\Rightarrow$ -1 and 1 have same image. $\therefore$ f is not injective.

There are such numbers of co-domain which have no image in domain Z.

e.g. 3 $\in$ co-domain, but $\sqrt{3} \notin$ domain of f. $\therefore$ f is not surjective.

(iii) $f: R \to R$ given by $f(x) = x^2$

As $f(-1) = f(1) = 1$

$\Rightarrow$ -1 and 1 have same image. $\therefore$ f is not injective.

e.g. $-2 \in$ co-domain, but $\sqrt{-2} \notin$ domain R of f. $\therefore$ f is not surjective.

(iv) $f: N \to N$ given by $f(x) = x^3$

If $f(x_1) = f(x_2)$ then $x_1^3 = x_2^3$

$\Rightarrow$ $x_1 = x_2$

i.e., for every $x \in$ N, has a unique image in its co-domain. $\therefore$ f is injective.

There are many such members of co-domain of f which do not have pre-image in its domain e.g., 2, 3, etc.

Therefore f is not onto. $\therefore$ f is not surjective.

(v) $f: Z \to Z$ given by $f(x) = x^3$

If $f(x_1) = f(x_2)$ then $x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

i.e., for every $x \in$ Z, has a unique image in its co-domain. $\therefore f$ is injective.

There are many such members of co-domain of f which do not have pre-image in its domain.

Therefore f is not onto. f is not surjective.

---

3. Prove that the Greatest integer Function $f: \mathrm{R} \to \mathrm{R}$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Ans. Function $f: \mathrm{R} \to \mathrm{R}$, given by $f(x) = [x]$

$\because 1 \le x \le 2$, $f(x) = 1$

$\therefore f(1) = 1$ and $f(1.1) = 1$

$\therefore f$ is not one-one.

All the images of $x \notin$ R belong to its domain have integers as the images in co-domain. But no fraction proper or improper belonging to co-domain of f has any pre-image in its domain.

Therefore, f is not onto.

---

4. Show that the Modulus Function $f: \mathrm{R} \to \mathrm{R}$, given by $f(x) = |x|$, is neither one-one nor onto, where is x, if x is positive or 0 and $|x|$ is -1, if x is negative.

Ans. Modulus Function $f: \mathrm{R} \to \mathrm{R}$, given by $f(x) = |x|$

Now $|x| = \begin{cases} -x, & \text{if } x < 0 \\ 0, & \text{if } x = 0 \\ x, & \text{if } x \geq 0 \end{cases}$

$\Rightarrow f$ contains $(-1,1), (1,1), (-2,2), (2,2)$

Thus negative integers are not images of any element. $\therefore f$ is not one-one.

Also second set R contains some negative numbers which are not images of any real number.

$\therefore f$ is not onto.

---

5. Show that the Signum Function $f: \mathrm{R} \to \mathrm{R}$, given by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$ is neither one-one nor onto.

Ans. Signum Function $f: \mathrm{R} \to \mathrm{R}$, given by $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

$f(1) = f(2) = 1$

$\Rightarrow f(x_1) = f(x_2) = 1$ for $n > 0$

$\Rightarrow x_1 \neq x_2 \quad \therefore f$ is not one-one.

Except $-1, 0, 1$ no other members of co-domain of f has any pre-image its domain.

$\therefore f$ is not onto.

Therefore, f is neither one-one nor onto.

---

6. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

Ans. A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)}

Here, $f(1)=4$, $f(2)=5$ and $f(3)=6$

Here, also distinct elements of A have distinct images in B.

Therefore, f is a one-one function.

---

7. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) $f:\mathbb{R}\to\mathbb{R}$ **defined by** $f(x)=3-4x$

(ii) $f:\mathbb{R}\to\mathbb{R}$ **defined by** $f(x)=1+x^2$

Ans. (i) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=3-4x$

Now, if $x_1, x_2 \in$ R, then $f(x_1)=3-4x_1$ and $f(x_2)=3-4x_2$

And if $f(x_1)=f(x_2)$, then $x_1=x_2$ $\therefore$ f is one-one.

Again, if every element of Y (– R) is image of some element of X (R) under f, i.e., for every $y\in$ Y, there exists an element x in X such that $f(x)=y$.

Now $y=3-4x$

$\Rightarrow x=\dfrac{3-y}{4}$

$\therefore f\left(\dfrac{3-y}{4}\right)=3-4\left(\dfrac{3-y}{4}\right)$

$\Rightarrow f(x)=3-3+y=y$

$\therefore$ f is onto or bijective function.

(ii) $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 1 + x^2$

Now, if $x_1, x_2 \in$ R, then $f(x_1) = 1 + x_1^2$ and $f(x_2) = 1 + x_2^2$

And if $f(x_1) = f(x_2)$, then $x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2 \therefore f$ is not one-one.

Again, if every element of Y (– R) is image of some element of X (R) under f, i.e., for every $y \in$ Y, there exists an element x in X such that $f(x) = y$.

Now, $y = 1 + x^2 \Rightarrow x = \pm\sqrt{y-1}$

$\therefore f\left(\sqrt{y-1}\right) = 1 + y - 1 = -y \neq y$

$\therefore f$ is not onto.

Therefore, f is not bijective.

---

8. Let A and B be sets. Show that f : A x B $\to$ B x A such that $f(a, b) = (b, a)$ is a bijective function.

Ans. Injectivity: Let (a_1, b_1) and $(a_2, b_2) \in$ A x B such that $f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$ for all $(a_1, b_1), (a_2, b_2) \in$ A x B

So, f is injective.

Surjectivity: Let (b, a) be an arbitrary element of B x A. Then $b \in$ B and $a \in$ A.

$\Rightarrow (a, b) \in$ A x B

Thus, for all $(b, a) \in$ B x A, their exists $(a, b) \in$ A x B such that $f(a, b) = (b, a)$

So, f : A x B $\rightarrow$ B x A is an onto function, therefore f is bijective.

---

9. Let $f: \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$ for all $n \in$ N.

State whether the function f is bijective. Justify your answer.

Ans. $f: \mathbb{N} \to \mathbb{N}$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$

(a) $f(1) = \frac{n+1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$ and $f(2) = \frac{n}{2} = \frac{2}{2} = 1$

The elements 1, 2, belonging to domain of f have the same image 1 in its co-domain.

So, f is not one-one, therefore, f is not injective.

(b) Every number of co-domain has pre-image in its domain e.g., 1 has two pre-images 1 and 2.

So, f is onto, therefore, f is not bijective.

---

10. Let A = R – {3} and B = R – {1}. Consider the function f : A $\rightarrow$ B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Ans. A = R – {3} and B = R – {1} and $f(x) = \frac{x-2}{x-3}$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1 - 2}{x_1 - 3}$ and $f(x_2) = \frac{x_2 - 2}{x_2 - 3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2 \therefore f$ is one-one function.

Now $y = \frac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{3y - 2 - 2y + 2}{3y - 2 - 3y + 3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

11. Let $f:\ \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^4$. Choose the correct answer:

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

Ans. $f(x) = x^4$ and $\underline{\mathbb{R}} \to \underline{\mathbb{R}}$

Let $x_1, x_2 \in \underline{\mathbb{R}}$, then $f(x_1) = x_1^4$ and $f(x_2) = x_2^4$

$\therefore\ x_1^4 = x_2^4$

$\Rightarrow\ \pm x_1 = \pm x_2$

Therefore, f is not one-one function.

Now, $y = x^4$

$\Rightarrow\ x = \pm y^{\frac{1}{4}}$

$\therefore\ f\left(y^{\frac{1}{4}}\right) = y^{\frac{1}{4}} = y$ and $f\left(-y^{\frac{1}{4}}\right) = -y^{\frac{1}{4}} = y$

Therefore, f is not onto function.

Therefore, option (D) is correct.

12. Let $f:\ \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = 3x$. Choose the correct answer:

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto

Ans. Let $x_1, x_2 \to$ R such that $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

Therefore, f is one-one function.

Now, consider $y \in$ R (co-domain of f) certainly $x = \frac{y}{3} \in$ R (domain of f)

Thus for all $y \in$ R (co-domain of f) there exists $x = \frac{y}{3} \in$ R (domain of f) such that

$$f(x) = f\left(\frac{y}{3}\right) = 3.\frac{y}{3} = y$$

Therefore, f is onto function.

Therefore, option (A) is correct.

CBSE Class–12 Mathematics
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Chapter - 1
Relations & Functions -Exercise 1.3

1. Let f : {1, 3, 4} $\to$ {1, 2, 5} and g : {1, 2, 5} $\to$ {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.

Ans. f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}

Now, $f(1)=2, f(3)=5, f(4)=1$ and $g(1)=3, g(2)=3, g(5)=1$

$(gof)(n)=g[f(x)]=g[f(1)]=g(2)=3$

$g[f(3)]=g(5)=1$ and $g[f(4)]=g(1)=3$

Hence, $gof=$ {(1, 3), (3, 1), (4, 3)}

2. Let f, g and h be functions from R $\longrightarrow$ R. Show that:

$(f+g)oh=foh+goh$

$(f.g)oh=(foh).(goh)$

Ans. (a) To prove: $(f+g)oh=foh+goh$

L. H. S. = $(f+g)oh=(f+g)[h(x)]$ = $f[h(x)]+g[h(x)]$ = $foh+goh$ = R. H. S.

(b) To prove: $(f.g)oh=(foh).(goh)$

L. H. S. = $(f.g)oh=(f.g)[h(x)]$ = $f[h(x)].g[h(x)]$ = $foh.goh$ = R. H. S.

3. Find gof and fog, if:

(i) $f(x)=|x|$ **and** $g(x)=|5x-2|$

(ii) $f(x)=8x^3$ **and** $g(x)=x^{\frac{1}{3}}$

Ans. To find: gof and fog

(i) $f(x)=|x|$ and $g(x)=|5x-2|$

$gof=g[f(x)]=g[|x|]$ and $fog=f[g(x)]=f[(5x-2)]$ = $|5x-2|$ = $|5|x|-2|$

(ii) $f(x)=8x^3$ and $g(x)=x^{\frac{1}{3}}$

$gof=g[f(x)]=g[8x^3]$ $(8x^3)^{\frac{1}{3}}=2x$

and $fog=f[g(x)]=f\left[\left(x^{\frac{1}{3}}\right)\right]$ = $8\left(x^{\frac{1}{3}}\right)^3$ $=8x$

4. If $f(x)=\dfrac{(4x+3)}{(6x-4)}, x\neq\dfrac{2}{3}$, **show that** $fof(x)=x$, **for all** $x\neq\dfrac{2}{3}$. **What is the inverse of** f?

Ans. Given: $f(x)=\dfrac{(4x+3)}{(6x-4)}, x\neq\dfrac{2}{3}$

L.H.S. = $fof(x)=f[f(x)]$ = $f\left[\dfrac{4x+3}{6x-4}\right]$ = $\dfrac{4\left(\dfrac{4x+3}{6x-4}\right)+3}{6\left(\dfrac{4x+3}{6x-4}\right)-4}$ = $\dfrac{16x+12+18x-12}{24x+18-24x+16}$

= $\dfrac{34x}{34}=x$ = R.H.S.

Now, $y=\dfrac{4x+3}{6x-4}$

$\Rightarrow 6xy-4y=4x+3$

$\Rightarrow 6xy-4x=4y+3$

$\Rightarrow x(6y-4)=4y+3$

$\Rightarrow x=\dfrac{4y+3}{6y-3}$

$\Rightarrow y=\dfrac{4x+3}{6x-3}$

Hence inverse of $f=f$.

5. State with reason whether following functions have inverse:

(i) f : {1, 2, 3, 4} $\rightarrow$ {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) g : {5, 6, 7, 8} $\rightarrow$ {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

(iii) h : {2, 3, 4, 5} $\rightarrow$ {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Ans. (i) f = {(1, 10), (2, 10), (3, 10), (4, 10)}

It is many-one function, therefore f has no inverse.

(ii) g = {(5, 4), (6, 3), (7, 4), (8, 2)}

It is many-one function, therefore g has no inverse.

(iii) h = {(2, 7), (3, 9), (4, 11), (5, 13)}

h is one-one onto function, therefore, h has an inverse.

6. Show that $f:[-1,1]\rightarrow$ R given by $f(x)=\frac{x}{(x+2)}$ is one-one. Find the inverse of the function $f:[-1,1]\rightarrow$ Range f.

Ans. Part I: $f:[-1,1]\rightarrow$ R given by $f(x)=\frac{x}{(x+2)}, x\neq-2$

Let $x_1, x_2\in[-1,1]$, then $f(x_1)=\frac{x_1}{x_1+2}$ and $f(x_2)=\frac{x_2}{x_2+2}$

When $f(x_1)=f(x_2)$ then $\frac{x_1}{x_1+2}=\frac{x_2}{x_2+2}$

$\Rightarrow x_1x_2+2x_1=x_1x_2+2x_2$

$\Rightarrow x_1=x_2 \therefore f$ is one-one.

Part II: Let $y\in$ Range of f

$\Rightarrow y=f(x)=\frac{x}{x+2}$ for some x in $[-1,1]$

As $y=\frac{x}{x+2}$

$\Rightarrow yx+2y=x$

$\Rightarrow x(1-y)=2y$

$\Rightarrow x=\frac{2y}{1-y}$

$\Rightarrow f^{-1}(y)=\frac{2y}{1-y} \therefore f$ is onto.

Therefore, $f^{-1}x=\frac{2x}{1-x}$

7. Consider f : R $\rightarrow$ R given by $f(x)=4x+3$. Show that f is invertible. Find the inverse of f. [Hint: $f^{-1}(y) = \frac{y-3}{4}$]

Ans. Consider f : R $\rightarrow$ R given by $f(x)=4x+3$

Let $x_1, x_2 \in$ R, then $f(x_1)=4x_1+3$ and $f(x_2)=4x_2+3$

Now, for $f(x_1)=f(x_2)$, then $4x_1+3=4x_2+3 \Rightarrow x_1=x_2 \;\therefore\; f$ is one-one.

Let $y \in$ Range of f

$\Rightarrow y=4x+3$

$\Rightarrow x=\dfrac{y-3}{4}$

$\therefore f\left(\dfrac{y-3}{4}\right)=4\left(\dfrac{y-3}{4}\right)+3=y$

$\Rightarrow f(x)=y \;\therefore\; f$ is onto.

Therefore, f is invertible and hence, $x=f^{-1}(y)=\dfrac{y-3}{4}$.

8. Consider f: $R_+ \rightarrow [4,\infty]$ given by $f(x)=x^2+4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y)=\sqrt{y-4}$, where R_+ is the set of all non-negative real numbers.

Ans. Consider f: $R_+ \rightarrow [4,\infty]$ and $f(x)=x^2+4$.

Let $x_1, x_2 \in$ R $\rightarrow [4,\infty]$, then $f(x_1)=x_1^2+4$ and $f(x_2)=x_2^2+4$

$\Rightarrow x_1^2+4=x_2^2+4$

$\Rightarrow x_1=x_2 \;\therefore\; f$ is one-one.

Now $y = x^2 + 4$

$\Rightarrow x = \sqrt{y-4}$ as $x > 0$

$\therefore f\left(\sqrt{y-4}\right) = \left(\sqrt{y-4}\right)^2 + 4 = y$

$\Rightarrow f(x) = y \therefore f$ is onto.

Therefore, $f(x)$ is invertible and $f^{-1}(y) = \sqrt{y-4}$.

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9. Consider $f: R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\left(\sqrt{y+6}\right)-1}{3}\right)$.

Ans. Consider $f: R_+ \to [-5, \infty]$ and $f(x) = 9x^2 + 6x - 5$.

Let $x_1, x_2 \in R \to [-5, \infty]$, then $f(x_1) = 9x_1^2 + 6x_1 - 5$ and $f(x_2) = 9x_2^2 + 6x_2 - 5$

Now, $f(x_1) = f(x_2)$ then $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9x_1^2 + 6x_1 = 9x_2^2 + 6x_2$

$\Rightarrow 9\left(x_1^2 - x_2^2\right) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)\left[9(x_1 + x_2) + 6\right] = 0$

$\Rightarrow x_1 - x_2 = 0$

$\Rightarrow x_1 = x_2 \therefore f$ is one-one.

Now, again $y = 9x^2 + 6x - 5$

$\Rightarrow 9x^2 + 6x - (5 + y) = 0$

$\Rightarrow \quad x = \dfrac{-6 \pm \sqrt{(6)^2 + 4 \times 9(5+y)}}{18} = \dfrac{-6 \pm 6\sqrt{1+5+y}}{18} = \dfrac{-6 \pm 6\sqrt{y+6}}{18} = \dfrac{\sqrt{y+6}-1}{3}$

$\therefore \; f(x) = f\left(\dfrac{\sqrt{y+6}-1}{3}\right) = 9\left(\dfrac{\sqrt{y+6}-1}{3}\right)^2 + 6\left(\dfrac{\sqrt{y+6}-1}{3}\right) - 5$

$= 9\left(\dfrac{y+6+1-2\sqrt{y+6}}{9}\right) + 2\left(\sqrt{y+6}-1\right) - 5$

$= y + 7 - 2\sqrt{y+6} + 2\sqrt{y+6} - 2 - 5 = y \quad \therefore \; f$ is onto.

Therefore, $f(x)$ is invertible and $f^{-1}(x) = \dfrac{\sqrt{y+6}-1}{3}$.

10. Let $f : X \rightarrow Y$ be an invertible function. Show that f has unique inverse.

(Hint: Suppose g_1 and g_2 are two inverses of f. Then for all $f \in Y, fog_1(y) = 1_Y(y) = fog_2(y)$. Use one-one ness of f).

Ans. Given: $f : X \rightarrow Y$ be an invertible function.

Thus f is 1 – 1 and onto and therefore f^{-1} exists.

Let g_1 and g_2 be two inverses of f. Then for all $y \in$ Y,

$fog_1(y) = I_y(y) = fog_2(y) \quad \therefore \; fog_1(y) = fog_2(y)$

$\Rightarrow f[g_1(y)] = f[g_2(y)]$

$\Rightarrow g_1(y) = g_2(y)$

$\therefore$ The inverse is unique and hence f has a unique inverse.

11. Consider f : {1, 2, 3} $\rightarrow \{a, b, c\}$ given by $f(1)=a, f(2)=b$ and $f(3)=c$. Find f^{-1} and show that $\left(f^{-1}\right)^{-1}=f$.

Ans. $f=\{(1, a),(2, b),(3, c)\}$, then it is clear that f is 1 – 1 and onto and therefore f^{-1} exists.

Also, $f^{-1}=\{(1, a),(b, 2),(c, 3)\}$ and $\left(f^{-1}\right)^{-1}=\{(1, a),(2, b),(3, c)\}=f$

Hence, $\left(f^{-1}\right)^{-1}=f$

12. Let f : X $\rightarrow$ Y be an invertible function. Show that the inverse of f^{-1} is f, i.e., $\left(f^{-1}\right)^{-1}=f$.

Ans. Let f : X $\rightarrow$ Y be an invertible function.

Then f is one-one and onto

$\Rightarrow g: y \rightarrow$ X where g is also one-one and onto such that

$gof(x)=\mathrm{I}_x$ and $fog(y)=\mathrm{I}_y$

$\Rightarrow g=f^{-1}$

Now, $f^{-1} o\left(f^{-1}\right)^{-1}=\mathrm{I}$ and $fo\left[f^{-1} o\left(f^{-1}\right)^{-1}\right]=fo\mathrm{I}$

$\Rightarrow\left[fof^{-1}\right] o\left(f^{-1}\right)^{-1}=f$

$\Rightarrow \mathrm{I} o\left(f^{-1}\right)^{-1}=f$

$\Rightarrow\left(f^{-1}\right)^{-1}=f$

13. If f : R $\rightarrow$ R given by $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$, then $fof(x)$ is:

(A) $x^{\frac{1}{3}}$

(B) x^3

(C) x

(D) $\left(3-x^3\right)$

Ans. f : R $\rightarrow$ R and $f(x)=\left(3-x^3\right)^{\frac{1}{3}}$

$\Rightarrow$ $f[f(x)]=\left[3-[f(x)]^3\right]^{\frac{1}{3}}$ = $\left[3-\left\{\left(3-x^3\right)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{3}}$

= $\left[3-\left(3-x^3\right)\right]^{\frac{1}{3}}$ = $\left(3-3+x^3\right)^{\frac{1}{3}}$ = x

Therefore, option (C) is correct.

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14. Let f **: R –** $\left\{\frac{-4}{3}\right\} \rightarrow$ **R be a function defined as** $f(x)=\frac{4x}{3x+4}$. **The inverse of** f **is the map** g **: Range of** $f \rightarrow \mathrm{R}-\left\{\frac{-4}{3}\right\}$ **given by:**

(A) $g(y)=\frac{3y}{3-4y}$

(B) $g(y)=\frac{4y}{4-3y}$

(C) $g(y)=\frac{4y}{3-4y}$

(D) $g(y)=\frac{3y}{4-3y}$

Ans. Given: $f : R - \left\{\frac{-4}{3}\right\} \rightarrow$ R and $f\left(x\right) = \frac{4x}{3x+4}$

Now, Range of $f \rightarrow \mathrm{R} - \left\{-\frac{4}{3}\right\}$

Let $y = f\left(x\right)$

$\therefore \quad y = \frac{4x}{3x+4}$

$\Rightarrow 3xy + 4y = 4x$

$\Rightarrow x\left(4-3y\right) = 4y$

$\Rightarrow x = \frac{4y}{4-3y}$

$\therefore$ $f^{-1}(y) = g(y) = \frac{4y}{4-3y}$

Therefore, option (B) is correct.

CBSE Class–12 Mathematics

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Chapter - 1

Relations & Functions - Exercise 1.4

1. Determine whether or not each of the definition of * given below gives a binary operation. In the event that * is not a binary operation, given justification for this.

(i) On Z^+, define * by $a * b = a - b$

(ii) On Z^+, define * by $a * b = ab$

(iii) On R, define * by $a * b = ab^2$

(iv) On Z^+, define * by $a * b = |a - b|$

(v) On Z^+, define * by $a * b = a$

Ans. (i) On Z^+ = {1, 2, 3, ...}, $a * b = a - b$

Let $a = 1, b = 3$ $\therefore a * b = 1 - 3 = -2 \notin Z^+$

Therefore, operation * is not a binary operation on Z^+.

(ii) On Z^+ = {1, 2, 3, ...}, $a * b = ab$

Let $a = 2, b = 4$ $\therefore$ $a * b = 2 \times 4 = 8 \in Z^+$

Therefore, operation * is a binary operation on Z^+.

(iii) on R (set of real numbers) $a * b = ab^2$

Let $a = 5.2, b = 3$ $\therefore a * b = 5.2(3)^2 = 36.8 \in$ R

Therefore, operation * is a binary operation on R.

(iv) On Z^+ = {1, 2, 3, ...}, $a * b = |a - b|$

Let $a = 3, b = 7$ $\therefore$ $a * b = |3 - 7| = |-4| = 4 \in Z^+$

Therefore, operation * is a binary operation on Z^+.

(v) On Z^+ = {1, 2, 3, ...}, $a * b = a$

Let $a = 5, b = 7$ $\therefore$ $a * b = 5 \times 7 = 35 \in Z^+$

Therefore, operation * is a binary operation on Z^+.

2. For each binary operation * defined below, determine whether * is commutative or associative:

(i) On Z, define $a * b = a - b$

(ii) On Q, define $a * b = ab + 1$

(iii) On Q, define $a * b = \frac{ab}{2}$

(iv) On Z, define $a * b = 2^{ab}$

(v) On Z, define $a * b = a^b$

(vi) On R – {– 1}, define $a * b = \frac{a}{b+1}$

Ans. (i) For commutativity: $a * b = a - b$ and $b * a = b - a$ = $-(a - b) \neq a * b$

For associativity: $a * (b * c) = a * (b - c)$ = $a - (b - c) = (a - b + c)$

Also, $(a * b) * c = (a - b) * c$ = $(a - b - c)$

$\therefore$ $a * (b * c) \neq (a * b) * c$

Therefore, the operation * is neither commutative nor associative.

(ii) For commutativity: $a * b = ab + 1$ and $b * a = ba + 1 = ab + 1 = a * b$

For associativity: $a*(b*c) = a*(bc+1) = a(bc+1)+1 = abc+a+1$

Also, $(a*b)*c = (ab+1)c+1 = abc+c+1$

$\therefore\ a*(b*c) \neq (a*b)*c$

Therefore, the operation * is commutative but not associative.

(iii) For commutativity: $a*b = \frac{ab}{2}$ and $b*a = \frac{ba}{2} = \frac{ab}{2} = a*b$

For associativity: $a*(b*c) = a*\left(\frac{bc}{2}\right) = \frac{abc/2}{2} = \frac{abc}{4}$

Also, $(a*b)*c = \left(\frac{ab}{2}\right)*c = \frac{abc/2}{2} = \frac{abc}{4}$

$\therefore\ a*(b*c) = (a*b)*c$

Therefore, the operation * is commutative and associative.

(iv) For commutativity: $a*b = 2^{ab}$ and $b*a = 2^{ba} = 2^{ab} = a*b$

For associativity: $a*(b*c) = a*2^{bc} = (2)$

Also, $(a*b)*c = (2^{ab})*2 = 2^{ab} \times c$

$\therefore\ a*(b*c) \neq (a*b)*c$

Therefore, the operation * is commutative but not associative.

(v) For commutativity: $a*b = a^b$ and $b*a = b^a$

$\Rightarrow\ a*b \neq b*a$

For associativity: $a*(b*c) = a*b^c = (a)^{b^c}$

Also, $(a*b)*c = (a^b)*c = a^{bc}$

$\therefore\ a*(b*c) \neq (a*b)*c$

Therefore, the operation * is neither commutative nor associative.

(vi) For commutativity: $a*b = \dfrac{a}{b+1}$ and $b*a = \dfrac{b}{a+1} \Rightarrow a*b \neq b*a$

For associativity: $a*(b*c) = a*\left(\dfrac{b}{c+1}\right) = \dfrac{a}{\frac{b}{c+1}+1} = \dfrac{a(c+1)}{b+c+1}$

Also, $(a*b)*c = \left(\dfrac{a}{b+1}\right)*c = \dfrac{{a}/{b+1}}{c+1/c} = \dfrac{a}{(b+1)(c+1)}$

$\therefore\ a*(b*c) \neq (a*b)*c$

Therefore, the operation * is neither commutative nor associative.

3. Consider the binary operation $\wedge$ on the set {1, 2, 3, 4, 5} defined by $a \wedge b = \min\{a, b\}$. Write the operation table of the operation $\wedge$.

Ans. Let A = {1, 2, 3, 4, 5} defined by $a \wedge b = \min\{a, b\}$ i.e., minimum of a and b.

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

4. Consider a binary operation * on the set {1, 2, 3, 4, 5} given by the following multiplication table (table 1.2).

(i) Compute (2 * 3) * 4 and 2 * (3 * 4)

(ii) Is * commutative?

(iii) Compute (2 * 3) * (4 * 5)

(Hint: Use the following table)

Table 1.2

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

Ans. (i) 2 * 3 = 1 and 3 * 4 = 1

Now (2 * 3) * 4 = 1 * 4 = 1 and 2 * (3 * 4) = 2 * 1 = 1

(ii) 2 * 3 = 1 and 3 * 4 = 1

2 * 3 = 3 * 2 and other element of the given set.

Hence the operation is commutative.

(iii) (2 * 3) * (4 * 5) = 1 * 1 = 1

5. Let *' be the binary operation on the set {1, 2, 3, 4, 5} defined by $a *' b =$ H.C.F. of a and b. Is the operation *' same as the operation * defined in Exercise 4 above? Justify your answer.

Ans. Let A = {1, 2, 3, 4, 5} and $a *' b =$ H.C.F. of a and b.

*'	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1

5	1	1	1	1	5

We observe that the operation *' is the same as the operation * in Exp.4.

6. Let * be the binary operation on N given by $a*b=$ L.C.M. of a and b. Find:

(i) 5 * 7, 20 * 16

(ii) Is * commutative?

(iii) Is * associative?

(iv) Find the identity of * in N.

(v) Which elements of N are invertible for the operation *?

Ans. $a*b=$ L.C.M. of a and b

(i) 5 * 7 = L.C.M. of 5 and 7 = 35

20 * 16 = L.C.M. of 20 and 16 = 80

(ii) $a*b=$ L.C.M. of a and b = L.C.M. of b and a = $b*a$

Therefore, operation * is commutative.

(iii) $a*(b*c)=a*(\text{L.C.M. of } b \text{ and } c)$ = $\text{L.C.M. of }(a \text{ and L.C.M. of } b \text{ and } c)$

= $\text{L.C.M. of } a, b \text{ and } c$

Similarly, $(a*b)*c=\text{L.C.M. of } a, b \text{ and } c$

Thus, $a*(b*c)=(a*b)*c$

Therefore, the operation is associative.

(iv) Identity of * in N = 1 because $a*1$ = L.C.M. of a and 1 = a

(v) Only the element 1 in N is invertible for the operation * because $1*\frac{1}{1}=1$

7. Is * defined on the set {1, 2, 3, 4, 5} by $a * b =$ L.C.M. of a and b a binary operation? Justify your answer.

Ans. Let A = {1, 2, 3, 4, 5} and $a *' b =$ L.C.M. of a and b.

*	1	2	3	4	5
1	1	2	3	4	5
2	2	2	6	4	10
3	3	x	3	12	15
4	4	4	12	4	20
5	5	x	15	20	5

Here, 2 * 3 = 6 $\notin$ A

Therefore, the operation * is not a binary operation.

8. Let * be the binary operation on N defined by $a * b =$ H.C.F. of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on N?

Ans. $a * b =$ H.C.F. of a and b.

(i) $a * b =$ H.C.F. of a and b = H.C.F. of b and a = $b * a$

Therefore, operation * is commutative.

(ii) $(a * b) * c = (\text{H.C.F. of } a \text{ and } b) * c$ = $\text{H.C.F. of } (\text{H.C.F. of } a \text{ and } b) \text{ and } c$

= $\text{H.C.F. of } a, b \text{ and } c = a * (b * c)$

Therefore, the operation is associative.

$1 * a = a * 1 \neq a$

Therefore, there does not exist any identity element.

9. Let * be a binary operation on the set Q of rational numbers as follows:

(i) $a*b=a-b$ **(ii)** $a*b=a^2+b^2$

(iii) $a*b=a+ab$ **(iv)** $a*b=(a-b)^2$

(v) $a*b=\frac{ab}{4}$ **(vi)** $a*b=ab^2$

Find which of the binary operations are commutative and which are associative.

Ans. (i) $a*b=a-b=-(b-a)=-b*a$ $\therefore$ operation * is not commutative.

$(a*b)*c=(a-b)*c=(a-b)-c=a-b-c$

And $a*(b*c)=a*(b-c)=a-(b-c)=a-b+c$

Here, $(a*b)*c\neq a*(b*c)$ $\therefore$ operation * is not associative.

(ii) $a*b=a^2+b^2=b^2+a^2=b*a$ $\therefore$ operation * is commutative.

$(a*b)*c=(a^2+b^2)*c=(a^2+b^2)+c^2=a^2+b^2+c^2$

And $a*(b*c)=a*(b^2+c^2)=a^2+(b^2+c^2)^2$

Here, $(a*b)*c\neq a*(b*c)$ $\therefore$ operation * is not associative.

(iii) $a*b=a+ab=a(1+b)$ and $b*a=b+ba=b(1+a)\neq a*b$

Therefore, operation * is not commutative.

$(a*b)*c=(a+ab)*c=(a+ab)+(a+ab)c$

And $a*(b*c)=a*(b+bc)=a+a(b+bc)$

Here, $(a*b)*c\neq a*(b*c)$ $\therefore$ operation * is not associative.

(iv) $a*b=(a-b)^2=(b-a)^2=b*a$ $\therefore$ operation * is commutative.

$(a*b)*c=(a-b)^2*c=\left[(a-b)^2-c\right]^2$

And $a*(b*c)=a*(b-c)^2=\left[a-(b-c)^2\right]^2$

Here, $(a*b)*c\neq a*(b*c)$ $\therefore$ operation * is not associative.

(v) $a*b=\frac{ab}{4}=\frac{ba}{4}=b*a$ $\therefore$ operation * is commutative.

$(a*b)*c=\frac{ab}{4}*c=\frac{\frac{ab}{4}c}{4}=\frac{abc}{16}$ And $a*(b*c)=a*\frac{bc}{4}=\frac{a\frac{bc}{4}}{4}=\frac{abc}{16}$

Here, $(a*b)*c=a*(b*c)$ $\therefore$ operation * is associative.

(vi) $a*b=ab^2$ and $b*a=ba^2\neq a*b$ $\therefore$ operation * is not commutative.

$(a*b)*c=(ab^2)*c=(ab^2)c^2=ab^2c^2$

And $a*(b*c)=a*(bc^2)=a(bc^2)^2=ab^2c^4$

Here, $(a*b)*c\neq a*(b*c)$ $\therefore$ operation * is not associative.

10. Show that none of the operations given above the identity.

Ans. Let the identity be I.

(i) $a*\text{I}=a-\text{I}\neq a$

(ii) $a*\text{I}=a^2-\text{I}^2\neq a$

(iii) $a*\text{I}=a+a\text{I}\neq a$

(iv) $a*\text{I}=(a-\text{I})^2\neq a$

(v) $a * \text{I} = \frac{a\text{I}}{4} \neq a$

(vi) $a * \text{I} = a\text{I}^2 \neq a$

Therefore, none of the operations given above has identity.

11. Let A = N x N and * be the binary operation on A defined by $(a,b)*(c,d) = (a+c, b+d)$

Show that * is commutative and associative. Find the identity element for * on A, if any.

Ans. A = N x N and * is a binary operation defined on A.

$(a,b)*(c,d) = (a+c, b+d) = (c+a, d+b) = (c,d)*(a,b)$ $\therefore$ The operation is commutative

Again, $[(a,b)*(c,d)]*(e,f) = (a+c, b+d)*(e,f) = (a+c+e, b+d+f)$

And $(a,b)[(c,d)*(e,f)] = (a,b)*(c+e, e+f) = (a+c+e, b+d+f)$

Here, $[(a,b)*(c,d)]*(e,f) = (a,b)[(c,d)*(e,f)]$ $\therefore$ The operation is associative.

Let identity function be (e,f), then $(a,b)*(e,f) = (a+e, b+f)$

For identity function $a = a+e \Rightarrow e = 0$

And for $b+f = b \Rightarrow f = 0$

As 0 $\neq$ N, therefore, identity-element does not exist.

12. State whether the following statements are true or false. Justify:

(i) For an arbitrary binary operation * on a set N, $a*a = a \forall a \in \text{N}$.

(ii) If * is a commutative binary operation on N, then $a*(b*b) = (c*b)*a$.

Ans. (i) * being a binary operation on N, is defined as $a*a=a\forall a\in\mathrm{N}.$

Hence operation * is not defined, therefore, the given statement is false.

(ii) * being a binary operation on N.

$\therefore\ c*b=b*c\ \ \therefore\ (c*b)*a=(b*c)*a=a*(b*c)$

Thus, $a*(b*b)=(c*b)*a$, therefore the given statement is true.

13. Consider a binary operation * on N defined as $a*b=a^3+b^3$. Choose the correct answer:

(A) Is * both associative and commutative?

(B) Is * commutative but not associative?

(C) Is * associative but not commutative?

(D) Is * neither commutative nor associative?

Ans. $a*b=a^3+b^3=b^3+a^3=b*a$ $\therefore$ The operation * is commutative.

Again, $(a*b)*c=a*(a^3+b^3)=a^3(a^3+b^3)^3$

And $(a*b)*c=(a^3+b^3)*c=(a^3+b^3)^3+c^3\neq a*(b*c)$

$\therefore$ The operation * is not associative.

Therefore, option (B) is correct.

CBSE Class–12 Mathematics

NCERT solution

Chapter - 1

Relations & Functions - Miscellaneous Exercise

1. Let $f: \mathrm{R} \rightarrow \mathrm{R}$ be defined as $f(x)=10x+7$. Find the function $g: \mathrm{R} \rightarrow \mathrm{R}$ such that $gof = fog = \mathrm{I}_{\mathrm{R}}$.

Ans. Given: $f(x)=10x+7$

Now $gof = g[f(x)]$ and $fog = f[g(x)] = 10g(x)+7$

$\Rightarrow 10g(x)+7 = \mathrm{I}_{\mathrm{R}}(x) = x$

$\Rightarrow g(x) = \dfrac{x-7}{10}$

2. Let $f: \mathrm{W} \rightarrow \mathrm{W}$ be defined as $f(n)=n-1$, if n is odd and $f(n)=n+1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

Ans. Given: $f: \mathrm{W} \rightarrow \mathrm{W}$ defined as $f(n)=\begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$

Injectivity: Let n, m be any two odd real numbers, then $f(n)=f(m)$

$\Rightarrow n-1=m-1$

$\Rightarrow n=m$

Again, let n, m be any two even whole numbers, then $f(n)=f(m)$

$\Rightarrow n+1=m+1$

$\Rightarrow n = m$

Is n is even and m is odd, then $n \neq m$

Also, if $f(n)$ odd and $f(m)$ is even, then $f(n) \neq f(m)$

Hence, $n \neq m$

$\Rightarrow f(n) \neq f(m)$

$\therefore$ f is an injective mapping.

Surjectivity: Let n be an arbitrary whole number.

If n is an odd number, then there exists an even whole number $(n+1)$ such that

$f(n+1) = n+1-1 = n$

If n is an even number, then there exists an odd whole number $(n-1)$ such that

$f(n-1) = n-1+1 = n$

Therefore, every $n \in$ W has its pre-image in W.

So, $f : \mathrm{W} \rightarrow \mathrm{W}$ is a surjective. Thus f is invertible and f^{-1} exists.

For f^{-1}: $y = n-1$

$\Rightarrow n = y+1$ and $y = n+1 \Rightarrow n = y-1$

$$\therefore f^{-1}(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$$

Hence, $f^{-1}(y) = y$

3. If $f : \mathrm{R} \rightarrow \mathrm{R}$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

Ans. Given: $f(x) = x^2 - 3x + 2$

$\Rightarrow f[f(x)] = f(x^2 - 3x + 2)$

$\Rightarrow (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

= $x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

$\Rightarrow x^4 - 6x^3 + 10x^2 - 3x$

---

4. Show that the function $f : \mathrm{R} \rightarrow \{x \in \mathrm{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in$ R is one-one and onto function.

Ans. It is given that f: $\mathbf{R} \rightarrow \{x \in \mathbf{R} : -1 < x < 1\}$ is defined as f(x) = $\frac{x}{1+|x|}, x \in \mathbf{R}$

For one- one

Suppose f(x) = f(y), where x, y $\in$ **R**

$\Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$

It can be observed that if x is positive and y is negative.

Then, we have

$\frac{x}{1+x} = \frac{y}{1-y} \Rightarrow 2xy = x - y$

Since, x is positive and y is negative,

x > y$\Rightarrow x - y > 0$, but, 2xy is negative

Then, 2xy $\neq$ x - y.

Thus, the case of x being positive and y being negative can be ruled out.

Under a similar argument, x being negative and y being positive can also be ruled out.

$\therefore$ x and y have to be either positive or negative.

When x and y are both positive,we have

f(x) = f(y)$\Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$

When x and y are both negative,we have

f(x) =f(y) $\Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x - xy = y - xy \Rightarrow x = y$

$\therefore$ f is one - one

For Onto

Now, let y∈**R** such that -1< y <1.

If y is negative then, there exists x = $\frac{y}{1+y} \in \mathbf{R}$ such that

f(x) = $f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1+\left|\frac{y}{1+y}\right|} = \frac{\frac{y}{1+y}}{1+\left(\frac{-y}{1+y}\right)} = \frac{y}{1+y-y} = y$

If y is positive, then, there exists x = $\frac{y}{1-y} \in \mathbf{R}$ such that

f(x) = $f\left(\frac{y}{1-y}\right) = \frac{\left(\frac{y}{1-y}\right)}{1+\left|\frac{y}{1-y}\right|} = \frac{\frac{y}{1-y}}{1+\left(\frac{y}{1-y}\right)} = \frac{y}{1-y+y} = y$

Therefore, f is onto. Hence, f is one -one and onto.

5. Show that the function $f: \mathrm{R} \rightarrow \mathrm{R}$ given by $f(x) = x^3$ is injective.

Ans. Let $x_1, x_2 \in$ R be such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

Therefore, f is one-one function, hence $f(x) = x^3$ is injective.

6. Give examples of two functions $f: \mathrm{N} \rightarrow \mathrm{Z}$ and $\mathrm{g}: \mathrm{Z} \rightarrow \mathrm{Z}$ such that gof is injective but g is not injective.

(Hint: Consider $f(x) = x$ and $g(x) = |x|$)

Ans. Given: two functions $f: \mathrm{N} \rightarrow \mathrm{Z}$ and $\mathrm{g}: \mathrm{Z} \rightarrow \mathrm{Z}$

Let $f(x) = x$ and $g(x) = x \therefore (gof)(x) = f[f(x)] = g(x)$

Therefore, gof is injective but g is not injective.

7. Give examples of two functions $f: \mathrm{N} \rightarrow \mathrm{N}$ and $\mathrm{g}: \mathrm{N} \rightarrow \mathrm{N}$ such that gof is onto but f is not onto.

(Hint: Consider $f(x) = x+1$ **and** $g(x) = \begin{cases} x-1 \text{ if } x > 1 \\ 1 \text{ if } x = 1 \end{cases}$ **)**

Ans. Let $f(x) = x+1$

$$\therefore\ g(x) = \begin{cases} x-1 & \text{if } x > 1 \\ 1 & \text{if } x = 1 \end{cases}$$

These are two examples in which gof is onto but f is not onto.

---

8. Given a non empty set X, consider P (X) which is the set of all subsets of X.

Define the relation R in P (X) as follows:

For subsets A, B in P (X), ARB if and only if A $\subset$ B. Is R an equivalence relation on P (X)? Justify your answer.

Ans. (i) A $\subset$ A $\therefore$ R is reflexive.

(ii) A $\subset$ B $\neq$ B $\subset$ A $\therefore$ R is not commutative.

(iii) If A $\subset$ B, B $\subset$ C, then A $\subset$ C $\therefore$ R is transitive.

Therefore, R is not equivalent relation.

---

9. Given a non-empty set X, consider the binary operation * : P (X) x P (X) $\rightarrow$ P (X) given by A * B = A $\cap$ B $\forall$ A, B in P (X), where P (X) is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in P (X) with respect to the operation *.

Ans. Let S be a non-empty set and P(S) be its power set. Let any two subsets A and B of S.

$\Rightarrow$ A $\cup$ B $\subset$ S

$\Rightarrow$ A $\cup$ B $\in$ P(S)

Therefore, '$\cup$' is an binary operation on P(S).

Similarly, if A, B $\in$ P(S) and A – B $\in$ P(S), then the intersection of sets $\cap$ and difference of sets are also binary operation on P(S) and A $\cap$ S = A = S $\cap$ A for every subset A of sets

$\Rightarrow$ A $\cap$ S = A = S $\cap$ A for all A $\in$ P(S)

$\Rightarrow$ S is the identity element for intersection $(\cap)$ on P(S).

---

10. Find the number of all onto functions from the set {1, 2, 3, ……., n} to itself.

Ans. The number of onto functions that can be defined from a finite set A containing n elements onto a finite set B containing n elements = $2^n - n$.

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11. Let S = $\{a, b, c\}$ and T = {1, 2, 3}. Find F^{-1} of the following functions F from S to T, if it exists.

(i) F = $\{(a,3),(b,2),(c,1)\}$

(ii) F = $\{(a,2),(b,1),(c,1)\}$

Ans. S = $\{a, b, c\}$ and T = {1, 2, 3}

(i) F = $\{(a,3),(b,2),(c,1)\}$

$\Rightarrow F(a)=3, F(b)=2, F(c)=1$

$\Rightarrow F^{-1}(3)=a, F^{-1}(2)=b, F^{-1}(1)=c$

$\therefore F^{-1}=\{(3,a),(2,b),(1,c)\}$

(ii) $\{(a,2),(b,1),(c,1)\}$

F is not one-one function, since element b and c have the same image 1.

Therefore, F is not one-one function.

---

12. Consider the binary operation * : R x R $\rightarrow$ R and o = R x R $\rightarrow$ R defined as $a * b = |a-b|$ and $aob = a, \forall a, b \in$ R. Show that * is commutative but not associative, o is associative but not commutative. Further, show that $\forall, a, b, c \in$ R, $a*(boc) = (a*b)o(a*b)$. [If it is so, we say that the operation * distributes over the operation o]. Does o distribute over *? Justify your answer.

Ans. Part I: $a*b = |a-b|$ also $b*a = |b-a| = (a-b)$ $\therefore$ operation * is commutative.

Now, $a*(b*c) = a*|b-c| = |a-(b-c)| = |a-b+c|$

And $(a*b)*c = |a-b|*c = |a-b-c|$

Here, $a*(b*c) \neq (a*b)*c$ $\therefore$ operation * is not associative.

Part II: $aob = a \forall a, b \in$ R

And, $boa = b$

$\Rightarrow aob \neq boa$ $\therefore$ operation o is not commutative.

Now $ao(boc) = aob = a$ and $(aob)oc = aoc = a$

Here $ao(boc) = (aob)oc$ $\therefore$ operation o is associative.

Part III: L.H.S. $a*(boc)$ = $a*b = |a-b|$

R.H.S. $(a*b)o(a*b)$ = $(a-b)o(a-c) = |a-b|$ = L.H.S. Proved.

Now, another distribution law: $ao(b*c) = (aob)*(aob)$

L.H.S. $ao(b*c) = ao(|b-c|) = a$

R.H.S. $(aob)*(aob) = a*a = |a-a| = 0$

As L.H.S. $\neq$ R.H.S.

Therefore, the operation o does not distribute over.

13. Given a non-empty set X, let * : P (X) x P (X) $\rightarrow$ P (X) be defined as A * B = (A – B) $\cup$ (B – A), $\forall$ A, B $\in$ P (X). Show that the empty set ϕ is the identity for the operation * and all the elements A of P (X) are invertible with A^{-1} = A. (Hint: $(A-\phi)\cup(\phi-A)=A$ and $(A-A)\cup(A-A)=A*A=\phi$)

Ans. For every A $\in$ P(X), we have

$\phi * A = (\phi - A)\cup(A-\phi) = \phi \cup A = A$

And $A * \phi = (A-\phi)\cup(\phi - A) = A\cup\phi = A$

$\Rightarrow$ ϕ is the identity element for the operation * on P(X).

Also A * A = (A – A) $\cup$ (A – A) = $\phi\cup\phi=\phi$

$\Rightarrow$ Every element A of P(X) is invertible with A^{-1} = A.

14. Define binary operation * on the set {0, 1, 2, 3, 4, 5} as $a*b=\begin{cases} a+b, \text{if} a+b<6 \\ a+b-6 \text{if} a+b \geq 6 \end{cases}$

Show that zero is the identity for this operation and each element a of the set is invertible with $6-a$ being the inverse of a.

Ans. A binary operation (or composition) * on a (non-empty) set is a function * : A x A $\rightarrow$A. We denote $*(a,b)$ by $a*b$ for every ordered pair $(a,b)\in$ A x A.

$\Rightarrow$ A binary operation on a no-empty set A is a rule that associates with every ordered pair of elements a,b (distinct or equal) of A some unique element $a*b$ of A.

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0

2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

For all $a \notin$ A, we have $0 * a$ (mod 6) = 0

And $a * 1 = a(\bmod 6) = a$ and $a * 1 = a(\bmod 6) = a = 0$

$\Rightarrow$ 0 is the identity element for the operation.

Also on 0 = 0 – 0 = 0 *

2 * 1 = 3 = 1 * 2

$0^{-1} = 0 \quad 0^{-1} = 5$

15. Let A = {–1, 0, 1, 2}, B = {–4, –2, 0, 2} and $f, g:$ A $\rightarrow$ B be the functions defined by $f(x) = x^2 - x, x \in$ A and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in$ A. Are f and g equal? Justify your answer.

(Hint: One may note that two functions $f:$ A $\rightarrow$ B and $g:$ A $\rightarrow$ B such that $f(a) = g(a)\ \forall\ a \in$ A, are called equal functions).

Ans. When $x = -1$ then $f(x) = 1^2 + 1 = 2$ and $g(x) = 2\left|-1 - \frac{1}{2}\right| - 1 = 2$

At $x = 0,$ $f(0) = 0$ and $g(0) = 2\left|-\frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0$

At $x = 1,$ $f(1) = 1^2 - 1 = 0$ and $g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 2 \times \frac{1}{2} - 1 = 0$

At $x = 2$, $f(2) = 2^2 - 2 = 2$ and $g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = 3 - 1 = 2$

Thus for each $a \in$ A, $f(a) = g(a)$

Therefore, f and g are equal function.

---

16. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is:

(A) 1

(B) 2

(C) 3

(D) 4

Ans. It is clear that 1 is reflexive and symmetric but not transitive.

Therefore, option (A) is correct.

---

17. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is:

(A) 1
(B) 2
(C) 3
(D) 4

Ans. 2
Therefore, option (B) is correct.

---

18. Let $f: \mathrm{R} \to \mathrm{R}$ be the Signum Function defined as $\begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$ and $g: \mathrm{R} \to \mathrm{R}$ be the Greatest Function given by $g(x) = [x]$, where $[x]$ is greatest integer less than

or equal to x. Then, does fog and gof coincide in (0, 1)?

Ans. It is clear that $gof: \mathbb{R} \to \mathbb{R}$ and $fog: \mathbb{R} \to \mathbb{R}$

Consider $x = \frac{1}{2}$ which lie on (0, # 1)

Now, $(gof)\left(\frac{1}{2}\right) = g\left\{f\left(\frac{1}{2}\right)\right\} = g(1) = [1] = 1$

And $(fog)\left(\frac{1}{2}\right) = f\left\{g\left(\frac{1}{2}\right)\right\} = f\left(\left[\frac{1}{2}\right]\right) = f(0) = 0$

$\Rightarrow gof \neq fog$ in (0, 1]

No, fog and gof don't coincide in (0, 1].

---

19. Number of binary operation on the set $\{a, b\}$ are:

(A) 10

(b) 16

(C) 20

(D) 8

Ans. A = $\{a, b\}$

A x A = $\{(a, a), (a, b), (b, b)(b, a)\}$

$\therefore n(A \times A)$ = 4

Number of subsets = 2^4 = 16

Hence number of binary operation is 16.

Therefore, option (B) is correct.

CBSE Class–12 Mathematics

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Chapter - 2

Inverse Trigonometric Functions - Exercise 2.1

Find the principal values of the following:

1. $\sin^{-1}\left(\frac{-1}{2}\right)$

Ans. Let $\sin^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow \sin y = -\frac{1}{2}$

$\Rightarrow \sin y = -\sin\frac{\pi}{6}$

$\Rightarrow \sin y = \sin\left(-\frac{\pi}{6}\right)$

Since, the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

2. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$

$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos y = \cos \frac{\pi}{6}$

$\Rightarrow \ y = \frac{\pi}{6}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

3. $\cos ec^{-1}(2)$

Ans. Let $\cos ec^{-1}(2) = y$

$\Rightarrow \ \cos ec\ y = 2$

$\Rightarrow \ \cos ec\ y = \cos ec \frac{\pi}{6}$

Since, the principal value branch of $\cos ec^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Therefore, Principal value of $\cos ec^{-1}(2)$ is $\frac{\pi}{6}$.

4. $\tan^{-1}\left(-\sqrt{3}\right)$

Ans. Let $\tan^{-1}\left(-\sqrt{3}\right) = y$

$\Rightarrow \ \tan y = -\sqrt{3}$

$\Rightarrow \ \tan y = -\tan \frac{\pi}{3}$

$\Rightarrow \ \tan y = \tan\left(-\frac{\pi}{3}\right)$

Since, the principal value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, Principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

5. $\cos^{-1}\left(\frac{-1}{2}\right)$

Ans. Let $\cos^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow \cos y = -\frac{1}{2}$

$\Rightarrow \cos y = -\cos\frac{\pi}{3}$

$\Rightarrow \cos y = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is $\frac{2\pi}{3}$.

6. $\tan^{-1}(-1)$

Ans. Let $\tan^{-1}(-1) = y$

$\Rightarrow \tan y = -1$

$\Rightarrow \tan y = -\tan\frac{\pi}{4}$

$\Rightarrow \tan y = \tan\left(-\frac{\pi}{4}\right)$

Since, the principal value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Therefore, Principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

---

7. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Ans. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=y$

$\Rightarrow \sec y=\frac{2}{\sqrt{3}}$

$\Rightarrow \sec y=\sec\frac{\pi}{6}$

Since, the principal value branch of $\sec^{-1}$ is $[0,\pi]$.

Therefore, Principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

---

8. $\cot^{-1}\left(\sqrt{3}\right)$

Ans. Let $\cot^{-1}\left(\sqrt{3}\right)=y$

$\Rightarrow \cot y=\sqrt{3}$

$\Rightarrow \cot y=\cot\frac{\pi}{6}$

Since, the principal value branch of $\cot^{-1}$ is $[0,\pi]$.

Therefore, Principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

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9. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

Ans. Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)=y$

$\Rightarrow \cos y=-\frac{1}{\sqrt{2}}$

$\Rightarrow \cos y=-\cos\frac{\pi}{4}$

$\Rightarrow \cos y=\cos\left(\pi-\frac{\pi}{4}\right)=\cos\frac{3\pi}{4}$

Since, the principal value branch of $\cos^{-1}$ is $[0,\pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

10. $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Ans. Let $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)=y$

$\Rightarrow \operatorname{cosec} y=-\sqrt{2}$

$\Rightarrow \operatorname{cosec} y=\operatorname{cosec}\frac{-\pi}{4}$

Since, the principal value branch of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Therefore, Principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is $\frac{-\pi}{4}$.

Find the value of the following:

11. $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

Ans. $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

$= \tan^{-1}\tan\frac{\pi}{4}+\cos^{-1}\left(-\cos\frac{\pi}{3}\right)+\sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$=\frac{\pi}{4}+\cos^{-1}\left(\cos\left(\pi-\frac{\pi}{3}\right)\right)+\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$

$= \frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$

$= \frac{3\pi+8\pi-2\pi}{12} = \frac{9\pi}{12}=\frac{3\pi}{4}$

---

12. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Ans. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

$= \cos^{-1}\cos\frac{\pi}{3}+2\sin^{-1}\sin\frac{\pi}{6}$

$= \frac{\pi}{3}+2\left(\frac{\pi}{6}\right)$

$= \frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

---

13. If $\sin^{-1}x=y,$ **then:**

A) $0\le y\le\pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Ans. By definition of principal value for $y = \sin^{-1} x,\ -\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Therefore, option (B) is correct.

---

14. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ **is equal to:**

(A) π

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Ans. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \tan^{-1}\tan\frac{\pi}{3} - \sec^{-1}\left(-\sec\frac{\pi}{3}\right)$

$= \frac{\pi}{3} - \sec^{-1}\sec\left(\pi - \frac{\pi}{3}\right)$

$= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Therefore, option (B) is correct.

CBSE Class–12 Mathematics

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Chapter - 2

Inverse Trigonometric Functions - Exercise 2.2

Prove the following:

1. $3\sin^{-1}x = \sin^{-1}\left(3x-4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Ans. We know that: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

Putting $\sin\theta = x$

$\Rightarrow \theta = \sin^{-1}x$

$\therefore \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$\Rightarrow \sin 3\theta = 3x - 4x^3$

$\Rightarrow 3\theta = \sin^{-1}\left(3x-4x^3\right)$

Putting $\theta = \sin^{-1}x$,

$3\sin^{-1}x = \sin^{-1}\left(3x-4x^3\right)$

Proved.

2. $3\cos^{-1}x = \cos^{-1}\left(4x^3-3x\right), x \in \left[\frac{1}{2}, 1\right]$

Ans. We know that: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

Putting $\cos\theta = x$

$\Rightarrow \theta = \cos^{-1}x$

$\therefore \cos 3\theta = 4x^3 - 3x$

$\Rightarrow 3\theta = \cos^{-1}\left(3x-4x^3\right)$

Putting $\theta = \cos^{-1} x$,

$3\cos^{-1} x = \cos^{-1}\left(4x^3 - 3x\right)$ Proved.

3. $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Ans. L.H.S. = $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

= $\tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}$ $\left[\because \tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}\right]$

= $\tan^{-1}\frac{48+77}{264-14}$

= $\tan^{-1}\frac{125}{250}$

= $\tan^{-1}\frac{1}{2}$

= R.H.S.

Proved.

4. $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Ans. L.H.S. = $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\dfrac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+\tan^{-1}\dfrac{1}{7}$

$\left[\because 2\tan^{-1}x=\tan^{-1}\dfrac{2x}{1-x^2}\right]$

$= \tan^{-1}\dfrac{4}{3}+\tan^{-1}\dfrac{1}{7}$

$= \tan^{-1}\dfrac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}$

$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\dfrac{x+y}{1-xy}\right]$

$= \tan^{-1}\dfrac{28+3}{21-4}$

$= \tan^{-1}\dfrac{31}{17}$ = R.H.S.

Proved.

Write the following functions in the simplest form:

5. $\tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}, x\neq 0$

Ans. Putting $x=\tan\theta$ so that $\theta=\tan^{-1}x$

$\Rightarrow \tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$

$= \tan^{-1}\dfrac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$

$= \tan^{-1}\dfrac{\sec\theta-1}{\tan\theta}$

$= \tan^{-1}\left(\dfrac{\dfrac{1}{\cos\theta}-1}{\dfrac{\sin\theta}{\cos\theta}}\right)$

$= \tan^{-1}\left(\dfrac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\dfrac{2\sin^2\dfrac{\theta}{2}}{2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}\right)$

$= \tan^{-1}\left(\tan\dfrac{\theta}{2}\right)$

$= \dfrac{\theta}{2} = \dfrac{1}{2}\tan^{-1}x$

---

6. $\tan^{-1}\dfrac{1}{\sqrt{x^2-1}}, |x|>1$

Ans. Putting $x=\sec\theta$ so that $\theta=\sec^{-1}x$

$\Rightarrow \tan^{-1}\dfrac{1}{\sqrt{x^2-1}}$

$= \tan^{-1}\dfrac{1}{\sqrt{\sec^2\theta-1}}$

$= \tan^{-1} \frac{1}{\sqrt{\tan^2\theta}}$

$= \tan^{-1}\left(\frac{1}{\tan\theta}\right)$

$= \tan^{-1}(\cot\theta)$

$= \tan^{-1}\tan\left(\frac{\pi}{2}-\theta\right)$

$= \frac{\pi}{2}-\theta = \frac{\pi}{2}-\sec^{-1}x$

---

7. $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}, x<\pi$

Ans. $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

$= \tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$

$= \tan^{-1}\tan\frac{x}{2}$

$= \frac{x}{2}$

---

8. $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), 0<x<\pi$

Ans. $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Dividing the numerator and denominator by $\cos x$,

$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$

$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)$

$= \tan^{-1}\tan\left(\frac{\pi}{4}-x\right)$ $\left[\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x.\tan y}\right]$

$= \frac{\pi}{4}-x$

---

9. $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x|<a$

Ans. Putting $x=a\sin\theta$ so that $\theta=\sin^{-1}\frac{x}{a}$

$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\left(1-\sin^2\theta\right)}}\right)$

$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\cos^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{a\sin\theta}{a\cos\theta}\right)$

$= \tan^{-1}\tan\theta$

$= \theta = \sin^{-1}\dfrac{x}{a}$

---

10. $\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right), a>0, \left(-\dfrac{a}{\sqrt{3}} \le x \le \dfrac{a}{\sqrt{3}}\right)$

Ans. $\tan^{-1}\left(\dfrac{3a^2x-x^3}{a^3-3ax^2}\right)$

$= \tan^{-1}\left(\dfrac{3\left(\dfrac{x}{a}\right)-\left(\dfrac{x}{a}\right)^3}{1-3\left(\dfrac{x}{a}\right)^2}\right)$ [Dividing numerator and denominator by a^3]

Putting $\dfrac{x}{a} = \tan\theta$ so that $\theta = \tan^{-1}\dfrac{x}{a}$

$= \tan^{-1}\left(\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$= \tan^{-1}\tan 3\theta$

$= 3\theta = 3\tan^{-1}\dfrac{x}{a}$

Find the values of each of the following:

11. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\dfrac{1}{2}\right)\right]$

Ans. $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\sin^{-1}\sin\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2\times\frac{1}{2}\right]$

$= \tan^{-1}1$

$= \tan^{-1}\tan\frac{\pi}{4} = \frac{\pi}{4}$

---

12. $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

Ans. $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

$= \cot\frac{\pi}{2} = 0 \left[\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$

---

13. $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right], |x|<1, y>0 \text{ and } xy<1$

Ans. Putting $x=\tan\theta$ and $y=\tan\phi$

$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$= \tan\frac{1}{2}\left[\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}+\cos^{-1}\frac{1-\tan^2\phi}{1+\tan^2\phi}\right]$

$= \tan\frac{1}{2}\left[\sin^{-1}\sin 2\theta+\cos^{-1}\cos 2\phi\right]$

$= \tan\frac{1}{2}\left[2\theta+2\phi\right]$

$= \tan\left[\theta+\phi\right]$

$= \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

$= \frac{x+y}{1-xy}$

14. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, **then find the value of** x.

Ans. Given: $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1=\sin\frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$

$\Rightarrow \cos^{-1}x=\frac{\pi}{2}-\sin^{-1}\frac{1}{5}$

$\Rightarrow \cos^{-1}x=\cos^{-1}\frac{1}{5}\left[\because \sin^{-1}t+\cos^{-1}t=\frac{\pi}{2}\right]$

$\Rightarrow x=\frac{1}{5}$

15. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, **then find the value of** x.

Ans. Given: $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}=\frac{\pi}{4}\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$\Rightarrow \tan^{-1}\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x-1)(x+1)}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{x^2+2x-x-2+x^2-2x+x-2}{x^2-4-\left(x^2-1\right)}=\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{x^2-4-x^2+1}=\tan\frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2-4}{-3}=1$$

$$\Rightarrow 2x^2-4=-3$$

$$\Rightarrow 2x^2=1$$

$$\Rightarrow x^2=\frac{1}{2}$$

$$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

Find the values of each of the expressions in Exercises 16 to 18.

16. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Ans. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$= \sin^{-1}\left(\sin\frac{3\pi-\pi}{3}\right)$

$= \sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right]$

$= \sin^{-1}\sin\frac{\pi}{3}$

$= \frac{\pi}{3}$

17. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Ans. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$= \tan^{-1}\left(\tan\frac{4\pi-\pi}{4}\right)$

$= \tan^{-1}\left[\tan\left(\pi-\frac{\pi}{4}\right)\right]$

$= \tan^{-1}\left[-\tan\frac{\pi}{4}\right]$

$= \tan^{-1}\tan\left(-\frac{\pi}{4}\right)$

$= -\frac{\pi}{4}$

18. $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Ans. Putting $\sin^{-1}\frac{3}{5}=x$ and $\cot^{-1}\frac{3}{2}=y$ so that $\sin x=\frac{3}{5}$ and $\cot y=\frac{3}{2}$

Now, $\cos x=\sqrt{1-\sin^2 x}$ = $\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$

And $\tan x=\frac{\sin x}{\cos x}=\frac{3}{4}$

and $\tan y=\frac{1}{\cot y}=\frac{2}{3}$

$\therefore \tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

= $\tan(x+y)$

= $\frac{\tan x+\tan y}{1-\tan x\tan y}$ = $\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}$

= $\frac{\frac{17}{12}}{\frac{1}{2}}$ = $\frac{17}{6}$

---

19. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ **is equal to:**

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$

Ans. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

= $\cos^{-1}\left[\cos\left(2\pi-\frac{7\pi}{6}\right)\right]\left[\because \cos(2\pi-\theta)=\cos\theta\right]$

= $2\pi-\frac{7\pi}{6}=\frac{12\pi-7\pi}{6}=\frac{5\pi}{6}$

Therefore, option (B) is correct.

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20. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ **is equal to:**

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

Ans. $\sin^{-1}\left(-\frac{1}{2}\right)=\sin^{-1}\left(-\sin\frac{\pi}{6}\right)=\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$

= $-\frac{\pi}{6}$

$\therefore \sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

$= \sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right]$
$= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$
$= \sin\frac{3\pi}{6} = \sin\frac{\pi}{2} = 1$
Therefore, option (D) is correct.

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21. $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$ **is equal to:**

(A) π

(B) $-\frac{\pi}{2}$

(C) 0

(D) $2\sqrt{3}$

Ans. $\tan^{-1}\sqrt{3} - \cot^{-1}\left(-\sqrt{3}\right)$
$= \tan^{-1}\tan\frac{\pi}{3} - \cot^{-1}\left(-\cot\frac{\pi}{6}\right)$
$= \frac{\pi}{3} - \cot^{-1}\left[\cot\left(\pi - \frac{\pi}{6}\right)\right]$
$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$
$= \frac{\pi}{3} - \frac{5\pi}{6}$
$= \frac{2\pi - 5\pi}{6}$
$= -\frac{3\pi}{6} = -\frac{\pi}{2}$
Therefore, option (B) is correct.

CBSE Class–12 Mathematics

NCERT solution

Chapter - 2

Inverse Trigonometric Functions - Miscellaneous Exercise

Find the value of the following:

1. $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Ans. $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$= \cos^{-1}\left(\cos\frac{12\pi+\pi}{6}\right)$

$= \cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right) = \frac{\pi}{6}$

2. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Ans. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$= \tan^{-1}\left(\tan\frac{6\pi+\pi}{6}\right)$

$= \tan^{-1}\left[\tan\left(\pi+\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right)$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$

3. Prove that: $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Ans. Let $\sin^{-1}\frac{3}{5} = \theta$ so that $\sin\theta = \frac{3}{5}$

$\therefore \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$

Since $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\times\frac{3}{4}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$

$\Rightarrow 2\theta = \tan^{-1}\frac{24}{7}$

$\Rightarrow 2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

4. Prove that: $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

Ans. Let $\sin^{-1}\frac{8}{17}=\theta$ so that $\sin\theta=\frac{8}{17}$

$$\therefore\ \cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\frac{64}{289}}=\sqrt{\frac{225}{289}}=\frac{15}{17}$$

$$\therefore\ \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{8}{15}$$

Again, Let $\sin^{-1}\frac{3}{5}=\phi$ so that $\sin\phi=\frac{3}{5}$

$$\therefore\ \cos\phi=\sqrt{1-\sin^2\phi}=\sqrt{1-\frac{9}{25}}=\sqrt{\frac{16}{25}}=\frac{4}{5}$$

$$\therefore\ \tan\phi=\frac{\sin\phi}{\cos\phi}=\frac{3}{4}$$

Since $\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

$$=\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}$$

$$=\frac{32+45}{60-24}=\frac{77}{36}$$

$$\Rightarrow\ \theta+\phi=\tan^{-1}\frac{77}{36}$$

$$\Rightarrow\ \sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$$

5. Prove that: $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

Ans. Let $\cos^{-1}\frac{4}{5}=\theta$ so that $\cos\theta=\frac{4}{5}$

$\therefore \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$

Again, Let $\cos^{-1}\frac{12}{13}=\phi$ so that $\cos\phi=\frac{12}{13}$

$\therefore \sin\phi=\sqrt{1-\cos^2\phi}=\sqrt{1-\frac{144}{169}}=\sqrt{\frac{25}{169}}=\frac{5}{13}$

Since $\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$ = $\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}$

= $\frac{48-15}{65}=\frac{33}{65}$

$\Rightarrow\ \theta+\phi=\cos^{-1}\frac{33}{65}$

$\Rightarrow\ \cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

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6. Prove that: $\cos^{-1}\frac{12}{13}+\sin^{-1}\frac{3}{5}=\sin^{-1}\frac{56}{65}$

Ans. Let $\cos^{-1}\frac{12}{13}=\theta$ so that $\cos\theta=\frac{12}{13}$

$\therefore \sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{144}{169}}=\sqrt{\frac{25}{169}}=\frac{5}{13}$

Again, Let $\sin^{-1}\frac{3}{5}=\phi$ so that $\sin\phi=\frac{3}{5}$

$\therefore \cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

Since $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi = \frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5}$

$= \frac{20+36}{65} = \frac{56}{65}$

$\Rightarrow \theta+\phi = \sin^{-1}\frac{56}{65}$

$\Rightarrow \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

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7. Prove that: $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

Ans. Let $\sin^{-1}\frac{5}{13} = \theta$ so that $\sin\theta = \frac{5}{13}$

$\therefore \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\therefore \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{5}{12}$

Again, Let $\cos^{-1}\frac{3}{5} = \phi$ so that $\cos\phi = \frac{3}{5}$

$\therefore \sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{4}{3}$

Since $\tan(\theta+\phi)=\dfrac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$ = $\dfrac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}$

= $\dfrac{\frac{21}{12}}{\frac{4}{9}}=\dfrac{63}{16}$

$\Rightarrow\ \theta+\phi=\tan^{-1}\dfrac{63}{16}$

$\Rightarrow\ \tan^{-1}\dfrac{63}{16}=\sin^{-1}\dfrac{5}{13}+\cos^{-1}\dfrac{3}{5}$

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8. Prove that: $\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{7}+\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{8}=\dfrac{\pi}{4}$

Ans. L.H.S. = $\left(\tan^{-1}\dfrac{1}{5}+\tan^{-1}\dfrac{1}{7}\right)+\left(\tan^{-1}\dfrac{1}{3}+\tan^{-1}\dfrac{1}{8}\right)$

= $\tan^{-1}\left(\dfrac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right)+\tan^{-1}\left(\dfrac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right)$

$\left[\because \tan^{-1}x+\tan^{-1}y=\tan^{-1}\dfrac{x+y}{1-xy}\right]$

= $\tan^{-1}\left(\dfrac{\frac{12}{35}}{\frac{34}{35}}\right)+\tan^{-1}\left(\dfrac{\frac{11}{24}}{\frac{23}{24}}\right)$

= $\tan^{-1}\dfrac{6}{17}+\tan^{-1}\dfrac{11}{23}$

= $\tan^{-1}\left(\dfrac{\dfrac{6}{17}+\dfrac{11}{23}}{1-\dfrac{6}{17}\times\dfrac{11}{23}}\right)$

= $\tan^{-1}\left(\dfrac{138+187}{391-66}\right)$

= $\tan^{-1}\dfrac{325}{325}=\tan^{-1}1=\dfrac{\pi}{4}$

R.H.S.

9. Prove that: $\tan^{-1}\sqrt{x}=\dfrac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right), x\in[0,1]$

Ans. Let $\tan^{-1}\sqrt{x}=\theta$ so that $\tan\theta=\sqrt{x}$

$\Rightarrow\ x=\tan^2\theta$

$\therefore\ \dfrac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$

= $\dfrac{1}{2}\cos^{-1}\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

= $\dfrac{1}{2}\cos^{-1}\cos 2\theta$

= $\dfrac{1}{2}\times 2\theta=\theta$

= $\tan^{-1}\sqrt{x}$

10. Prove that: $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\dfrac{x}{2}, x\in\left(0,\dfrac{\pi}{4}\right)$

Ans. We know that $1+\sin x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\cos\frac{x}{2}\sin\frac{x}{2}$ = $\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2$

Again, $1-\sin x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\cos\frac{x}{2}\sin\frac{x}{2}$

= $\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2$

$\therefore$ L.H.S. = $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

= $\cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$

= $\cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$

= $\cot^{-1}\cot\frac{x}{2}=\frac{x}{2}$

11. Prove that: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x, \frac{-1}{\sqrt{2}}\leq x\leq 1$

Ans. Putting $x=\cos 2\theta$ so that $\theta=\frac{1}{2}\cos^{-1}x$

L.H.S. = $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$

Dividing every term by $\sqrt{2}\cos\theta$,

$= \tan^{-1}\left(\dfrac{1-\tan\theta}{1+\tan\theta}\right)$

$= \tan^{-1}\left(\dfrac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)$

$= \tan^{-1}\tan\left(\dfrac{\pi}{4}-\theta\right) = \dfrac{\pi}{4}-\theta$

$= \dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}x$ = R.H.S.

---

12. Prove that: $\dfrac{9\pi}{8}-\dfrac{9}{4}\sin^{-1}\dfrac{1}{3}=\dfrac{9}{4}\sin^{-1}\dfrac{2\sqrt{2}}{3}$

Ans. L.H.S. = $\dfrac{9\pi}{8}-\dfrac{9}{4}\sin^{-1}\dfrac{1}{3}$

$= \dfrac{9}{4}\left(\dfrac{\pi}{2}-\sin^{-1}\dfrac{1}{3}\right)$

= $\frac{9}{4}\cos^{-1}\frac{1}{3}$ ...(i) $\left[\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}\right]$

Now, let $\theta = \cos^{-1}\frac{1}{3}$ so that $\cos\theta = \frac{1}{3}$

$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

$\Rightarrow \quad \theta = \sin^{-1}\frac{2\sqrt{2}}{3}$

$\therefore$ From eq. (i), $\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ = R.H.S.

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13. Solve the equation: $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

Ans. $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2}{\sin x}\right)$

$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = \frac{2}{\sin x}$

$\Rightarrow \frac{\cos x}{\sin x} = 1$

$\Rightarrow \cot x = 1$

$\Rightarrow x = \frac{\pi}{4}$

---

14. Solve the equation: $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x, (x > 0)$

Ans. Putting $x = \tan\theta$

$$\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$$

$$\Rightarrow \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)=\frac{1}{2}\tan^{-1}\tan\theta$$

$$\Rightarrow \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{\tan\frac{\pi}{4}+\tan\theta}\right)=\frac{1}{2}\theta$$

$$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\theta}{2}$$

$$\Rightarrow \frac{\pi}{4}-\theta=\frac{\theta}{2}$$

$$\Rightarrow \theta+\frac{\theta}{2}=\frac{\pi}{4}$$

$$\Rightarrow \frac{3\theta}{2}=\frac{\pi}{4}$$

$$\Rightarrow 12\theta=2\pi$$

$$\Rightarrow \theta=\frac{\pi}{6}$$

$$\therefore x=\tan\theta=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$$

15. $\sin\left(\tan^{-1}x\right), |x|<1$ **is equal to:**

(A) $\frac{x}{\sqrt{1-x^2}}$

(B) $\frac{1}{\sqrt{1-x^2}}$

(C) $\frac{1}{\sqrt{1+x^2}}$

(D) $\frac{x}{\sqrt{1+x^2}}$

Ans. Let $\sin\left(\tan^{-1}x\right)=\sin\theta$ where $\theta=\tan^{-1}x$ so that $x=\tan\theta$

$$\Rightarrow \sin\left(\tan^{-1}x\right)=\frac{1}{\cos ec\theta}=\frac{1}{\sqrt{1+\cot^2\theta}}$$

Putting $\cot\theta=\frac{1}{\tan\theta}=\frac{1}{x}$

$$\Rightarrow \sin\left(\tan^{-1}x\right)=\frac{1}{\sqrt{1+\frac{1}{x^2}}}=\frac{x}{\sqrt{x^2+1}}$$

Therefore, option (D) is correct.

16. $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, **then** x **is equal to:**

(A) $0, \frac{1}{2}$

(B) $1, \frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

Ans. Putting $\sin^{-1} x = \theta$ so that $x = \sin\theta$

$\therefore\ \sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}(1-x) - 2\theta = \frac{\pi}{2}$

$\Rightarrow\ \sin^{-1}(1-x) = \frac{\pi}{2} + 2\theta$

$\Rightarrow\ 1 - x = \sin\left(\frac{\pi}{2} + 2\theta\right)$

$\Rightarrow 1 - x = \cos 2\theta$

$\Rightarrow\ 1 - x = 1 - 2\sin^2\theta$

$\Rightarrow\ 1 - x = 1 - 2x^2\ [x = \sin\theta]$

$\Rightarrow\ -x = -2x^2$

$\Rightarrow 2x^2 - x = 0$

$\Rightarrow\ x(2x-1) = 0$

$\Rightarrow\ x = 0$ or $2x - 1 = 0$

$\Rightarrow\ x = 0$ or $x = \frac{1}{2}$

But $x = \frac{1}{2}$ does not satisfy the given equation.

Therefore, option (C) is correct.

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17. $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ **is equal to:**

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) $-\frac{3\pi}{4}$

Ans. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$

$= \tan^{-1}\left[\frac{\frac{x}{y}-\left(\frac{x-y}{x+y}\right)}{1+\frac{x}{y}\left(\frac{x-y}{x+y}\right)}\right]$

$= \tan^{-1}\left[\frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}\right]$

$= \tan^{-1}\left(\frac{x^2+xy-xy+y^2}{xy+y^2+x^2-xy}\right)$

$= \tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)$

$= \tan^{-1}1$

$= \tan^{-1}\tan\frac{\pi}{4}=\frac{\pi}{4}$

Therefore, option (C) is correct.